Short Paper: Data Detection Techniques for OFDM Signals over Doppler-Distorted Channels

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ABSTRACT

Signal scaling due to Doppler in underwater acoustic communications results in inter-carrier interference (ICI) and severely degraded detection performance in OFDM based systems in the absence of interference mitigation. Using a recently proposed partial FFT demodulation technique [5], where several FFT demodulators operate over parallel non-overlapping segments, two efficient algorithms which reduce ICI and error probability significantly are proposed. Numerical simulations demonstrate an improvement of several dB over conventional OFDM detection techniques at the cost of a small increase in complexity.

Categories and Subject Descriptors

H.4 [**Information Systems Applications**]: Communication Applications; C.3 [**Special-purpose and Application Based Systems**]: Signal Processing Systems

General Terms

Algorithms, Receiver Design, Underwater Communications

Keywords

OFDM, Doppler, Low complexity receivers, Partial FFT

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is the primary signaling scheme for several wireless radio communication systems and is being considered as the modulation of choice for underwater acoustic (UWA) communications (see [3] and the references therein). The latter are of special interest as they exemplify severely Doppler distorted channels. Interest in OFDM stems from the fact that it decomposes a static frequency selective channel into several flat channels, enabling low complexity equalization and symbol-by-symbol detection. However, OFDM is very sensitive to Doppler distortion (Doppler scaling in particular) which destroys subcarrier orthogonality and introduces severe inter-carrier interference (ICI).

Recent work has focused on two major approaches for demonstrating the viability of OFDM in UWA channels. The first is a pilot based, block oriented detection scheme [1] and assumes that the Doppler scaling can be estimated to a high degree of accuracy and compensated. The second approach considers the scenario where the Doppler scaling parameter is of the order of 10^{-5} and models the distortion as a progressively increasing phase shift across subcarriers in the frequency domain. Decision directed, adaptive block processing based algorithms to estimate and compensate for the modeled residual phase shift were proposed in [2](and the references therein). Another recently proposed technique does not take into account the signal structure due to Doppler scaling [4] and proposes both minimum mean square error and adaptive decision-feedback equalizers to compensate for the Doppler distortion. In this paper, we investigate data detection schemes using the pilot based approach.

We consider UWA communication links with Doppler scaling of the order of 10^{-4}. This may either be the result of residual error in estimating the resampling parameter [1] or slowly drifting transceivers in a stationary environment. In such scenarios, the effect of Doppler scaling cannot be modeled as a phase shift without a severe loss in detection performance. In our recent work on partial FFT demodulation [5], an algorithm which operates on partial OFDM intervals and combines the outputs was proposed and shown to achieve significant improvement in detection performance. The key challenge for the proposed technique was to appropriately design the combiner weights to reduce ICI.

In this paper, we propose two algorithms to compute the combiner weights for the partial FFT technique: recursive weight estimation across subcarriers and model based weight estimation. While the recursive estimator does not presume any knowledge of the channel variation and is in general applicable in a variety of scenarios, the model based weight estimator relies on a model of the distortion process. Numerical results show that partial FFT demodulation followed by weighted combining is effective in compensating Doppler shifts one order of magnitude larger than that by modeling it as a phase distortion. This approach allows us to achieve low symbol error rates in underwater environments at a moderate increase in detection complexity.

This paper is organized as follows. Section 2 presents the OFDM signal model and briefly illustrates the concept of partial FFT demodulation presented in [5]. Algorithms for data detection are presented in Section 3 and Section 4

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demonstrates the performance improvement that can be obtained for underwater acoustic channels. Section 5 concludes this paper.

2. SIGNAL MODEL

The transmitted OFDM signal in passband can be compactly written as

$$s(t) = \text{Re}\left[\sum_{l=1}^{K} d_l e^{j2\pi f_l t}\right], \quad t \in [-T_g, T] \qquad (1)$$

where d_k are the data symbols modulated onto the K subcarriers, $f_k = f_0 + (k-1)\Delta f$ is the frequency of the k^{th} subcarrier, f_0 is the frequency of the first subcarrier, T_g and T are the cyclic prefix and the OFDM symbol duration respectively and $\Delta f = 1/T$. The cyclic extension is assumed to be long enough to capture the effects of channel multipath and Doppler spreading. Assuming constant channel gains $h_p,\ p = 1, 2, \cdots, P$ and linearly varying path delays $\tau_p(t) = \tau_p - at$ over one OFDM symbol duration, the complex valued received signal can be expressed as

$$r(t) = \sum_{l=1}^{K} d_l H_l e^{j2\pi f_l t(1+a)} + n(t), \quad t \in [0, T] \qquad (2)$$

where $H_k = \sum_{p=1}^{P} h_p e^{-j2\pi f_k \tau_p}$ is the channel frequency response on the k^{th} carrier and $n(t)$ is additive white Gaussian noise (AWGN). The Doppler distortion due to time scaling is modeled with the parameter $a = v/c$, where v is the relative velocity between the transceivers and $c = 1500$m/sec is the speed of sound in water. Typical values of a are of the order of 10^{-3} for objects moving at a few meters per second (*e.g.*, at $v = 1.5$m/sec $\Rightarrow a = 10^{-3}$). The signal is first resampled at the receiver to compensate for the time scaling due to Doppler. However, inaccuracies in Doppler estimation result in residual time scaling. In this paper, the Doppler distortion parameter a is the residual signal scaling factor after resampling at the receiver and is typically on the order of 10^{-4}.

2.1 Partial FFT demodulation

The idea of partial FFT demodulation is illustrated in [5] (see this paper for a detailed discussion on this technique and some notation used below). At the receiver, the cyclic prefix is first discarded and the m^{th} partial FFT output at the k^{th} subcarrier is computed as

$$\begin{aligned} y_k(m) &= \int_{(m-1)T/M}^{mT/M} r(t) e^{-j2\pi f_k t} dt, \quad m = 1, 2, \cdots, M \\ &\approx e^{j2\pi a f_k \frac{2m-1}{2M} T} \sum_l d_l H_l I_{l-k}(m) + n_k(m). \end{aligned} \qquad (3)$$

where, $I_i(m) = \int_{(m-1)T/M}^{mT/M} e^{-j2\pi i \Delta f t}$ models the effect of partial integration. Let us define $\mathbf{y}_k = [y_k(1), \cdots, y_k(M)]^T$ as the vector of partial FFT outputs and $\mathbf{p}_k = [p_k(1), \cdots, p_k(M)]^T$ as the vector of weighting coefficients for the k^{th} subcarrier. After combining the M partial FFT outputs, data on the k^{th} subcarrier is

$$x_k = \sum_{m=1}^{M} p_k^*(m) y_k(m) = \mathbf{p}_k^H \mathbf{y}_k. \qquad (4)$$

When the distortion due to the Doppler scaling is perfectly compensated at the receiver using resampling, *i.e.* $a = 0$, the output received on the k^{th} subcarrier can be modeled as $x_k = H_k d_k + w_k$ as in a conventional OFDM system [1]. However, in the presence of residual Doppler, standard FFT processing at the receiver introduces considerable ICI. Partial FFT demodulation in this scenario allows for a considerable reduction in ICI by a judicious choice of the combiner weights $p_k(m)$ and allows the output of the k^{th} carrier to be approximated as $x_k \approx H_k d_k + n_k$.

In practice, the partial FFT is implemented using zero-padded inputs to conventional FFTs. Each FFT block is of size K, the same used at the transmitter, but operates on a windowed version of the signal samples. For example, the second FFT for an $M = 4$ system operates on the received signal vector for samples corresponding to the interval $[T/4, T/2]$; all others set to zero.

3. ALGORITHMS FOR DATA DETECTION

In this section, two new algorithms for computing the combiner weights are presented. The first algorithm is based on recursive weight estimation across subcarriers and does not presume any knowledge of the channel distortion. The second is a model based weight estimator which imposes a structure on the weights to compensate for the Doppler distortion and hence needs to be tailored to suit the distortion process. Both the algorithms are based on the assumption that the output after partial FFT combining can be modeled as

$$x_k \approx H_k d_k + w_k, \qquad (5)$$

where H_k is the channel frequency response and w_k contains the noise and residual ICI.

3.1 Recursive Weight Estimator

The recursive weight estimator is based on the assumption that the combiner weights $\mathbf{p}_k$ and the channel frequency response H_k are slowly varying with the subcarrier index k and typically true for OFDM systems having larger number of carriers for a fixed bandwidth. The algorithm operates in several steps. In the first step, the partial FFT outputs are combined to yield the signal $x_k = \mathbf{p}_k^H \mathbf{y}_k$. Next, using a estimate of the frequency response on the previous subcarrier, $\hat{H}_{k-1}$, the signal is equalized to form an estimate of the data symbol

$$\hat{d}_k = \frac{x_k}{\hat{H}_{k-1}}; \ \tilde{d}_k = \text{dec}(\hat{d}_k). \qquad (6)$$

The channel frequency response for the current carrier is then computed as

$$\hat{H}_k = \eta \hat{H}_{k-1} + \frac{x_k}{\tilde{d}_k}. \qquad (7)$$

Assuming correct symbol decisions, or using pilots when available, the error at the combiner output can be evaluated as

$$e_k = \hat{H}_k \tilde{d}_k - x_k. \qquad (8)$$

This error is then used to drive an adaptive algorithm for the combiner weights, for example the Recursive Least Squares (RLS) algorithm:

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \text{RLS}\left[y_k, e_k, \lambda, \delta\right]. \qquad (9)$$

[1] For a conventional FFT, $a = 0$ and $p_k(m) = 1, \ \forall k, m$

Pilot insertion beyond those necessary for initial convergence is required for channels that exhibit spectral nulls. On such channels, a subcarrier near a deep fade will experience symbol errors, which will propagate across subcarriers unless corrected [5]. To overcome the loss of detection performance due to error propagation, pilot symbols are inserted periodically throughout the OFDM symbol. Now, using the knowledge of the inserted periodic pilot symbols and the distortion compensated demodulator outputs x_k the channel is re-estimated as in a conventional OFDM system. This channel frequency response (as opposed to the one in (7)) is used to equalize and detect the data symbols from the computed demodulator outputs x_k. It has been observed that re-estimating the channel using already available pilots eliminates error propagation while recursively combining across subcarriers. The algorithm is summarized in Table 1.

3.2 Model Based Weight Estimator

The combiner weights are first parametrically modeled, the partial FFT outputs combined and the relevant parameters are then estimated when using the model based estimator. Let us define $[k_1, k_2, \cdots, k_P]$ to be the set of subcarriers carrying pilot symbols. The combiner weights are designed to compensate the phase distortion due to the residual Doppler scaling for each partial FFT demodulator. Let us define the diagonal matrix

$$\mathbf{D}_{\tilde{a}}(m) = \mathrm{diag}\left[e^{j2\pi f_{k_1}\tilde{a}\frac{2m-1}{2M}T}, \cdots, e^{j2\pi f_{k_P}\tilde{a}\frac{2m-1}{2M}T}\right],$$

as containing the phase due to the Doppler distortion on the pilot subcarriers in the m^{th} partial FFT interval. For a candidate value of the distortion parameter $\tilde{a}$, the partial FFT outputs for the pilot subcarriers are first combined:

$$\mathbf{x}^p_{\tilde{a}} = \sum_{m=1}^{M} \mathbf{D}^H_{\tilde{a}}(m)\mathbf{y}^p(m), \tag{10}$$

where the vector $\mathbf{y}^p(m) = \left[y^p_{k_1}(m), \cdots, y^p_{k_P}(m)\right]$. Now assuming that the $\mathbf{x}^p_{\tilde{a}}$ is free of Doppler distortion, it can be expressed as

$$\mathbf{x}^p_{\tilde{a}} = \mathbf{S}^p\mathbf{F}\mathbf{h} + \mathbf{w}^p, \quad \mathbf{F}_{m,l} = e^{j2\pi k_m l/K}, \tag{11}$$

where, $\mathbf{S}^p = \mathrm{diag}\left[d_{k_1}, d_{k_2}, \cdots, d_{k_P}\right]$. The channel impulse response for the candidate value of the Doppler scaling can be obtained as

$$\hat{\mathbf{h}}_{\tilde{a}} = \mathbf{F}^{\dagger}(\mathbf{S}^p)^{-1}\mathbf{x}^p_{\tilde{a}}. \tag{12}$$

The Doppler scaling can then be estimated using deterministic maximum likelihood method and is given as

$$\hat{a} = \arg\min_{\tilde{a}} \left|\mathbf{x}^p_{\tilde{a}} - \mathbf{S}^p\mathbf{F}\hat{\mathbf{h}}_{\tilde{a}}\right|^2 \tag{13}$$

$$= \arg\min_{\tilde{a}} \left|\mathbf{x}^p_{\tilde{a}} - \mathbf{S}^p\mathbf{F}\mathbf{F}^{\dagger}(\mathbf{S}^p)^{-1}\mathbf{x}^p_{\tilde{a}}\right|^2. \tag{14}$$

This minimization problem can be solved in a computationally efficient manner as it involves the projection of a vector over a pre-determined subspace. Using the estimated Doppler distortion parameter, the channel impulse response can be estimated as

$$\hat{\mathbf{h}} = \mathbf{F}^{\dagger}(\mathbf{S}^p)^{-1}\mathbf{x}^p_{\hat{a}}. \tag{15}$$

The data symbol detection is then carried out as in a conventional OFDM system which estimates the time-domain channel impulse response.

$$\tilde{d}_k = \mathrm{dec}(x_k/\hat{H}_k), \quad \hat{H}_k = \sum_{l=0}^{L-1} \hat{h}_l e^{-j2\pi kl/K}. \tag{16}$$

Algorithm 1 Recursive Weight Estimation

INITIALIZATION:
Weights: $\mathbf{p}_0 = [1, 1, \cdots, 1]^T_{M\times 1}$
covariance matrix: $\delta << 1$, $\mathbf{G}_0 = \delta^{-1}\mathbf{I}_{M\times M}$
Control parameters: $\lambda = 0.99$, $\eta = 0.2$
Channel estimates: $\hat{H}_0 = 1$
for $k = 1$ to K **do**
  COMPUTE SIGNALS:
  $\mathbf{y}_k = [y_1(k), \cdots, y_M(k)]^T$
  $x_k = \mathbf{p}^H_k\mathbf{y}_k$
  $\hat{d}_k = x_k/\hat{H}_{k-1}$
  DATA DETECTION/PILOTS:
  if $k \in \{k_1, k_2, \cdots, k_P\}$ **then**
    $\tilde{d}_k = d_k$
  else
    $\tilde{d}_k = \mathrm{dec}(\hat{d}_k)$, $\mathrm{dec}(\cdot)$ maps the point to the nearest constellation symbol.
  end if
  UPDATE THE CHANNEL:
  $\hat{H}_k = \eta\hat{H}_{k-1} + (1-\eta)x_k/\tilde{d}_k$
  UPDATE THE COMBINER (RLS ALGORITHM):
  $e_k = \hat{H}_k\tilde{d}_k - x_k$
  $\mathbf{g}_k = \frac{\mathbf{G}_k\mathbf{y}_k}{\lambda + \mathbf{y}^H_k\mathbf{G}_k\mathbf{y}_k}$
  $\mathbf{p}_{k+1} = \mathbf{p}_k + e^*_k\mathbf{g}_k$
  $\mathbf{G}_{k+1} = \frac{1}{\lambda}(\mathbf{G}_k - \mathbf{g}_k\mathbf{y}^H_k\mathbf{G}_k)$.
end for
RE-COMPUTE FREQUENCY RESPONSE:
  $\mathbf{x}^p = [x_{k_1}, x_{k_2}, \cdots, x_{k_P}]^T$
  $\mathbf{S}^p = \mathrm{diag}\left[d_{k_1}, d_{k_2}, \cdots, d_{k_P}\right]$
  $\hat{\mathbf{h}} = \mathbf{F}^{\dagger}(\mathbf{S}^p)^{-1}\mathbf{x}^p$, $\mathbf{F}_{m,l} = e^{j2\pi k_m l/K}$.
  $\hat{H}_k = \sum_{l=1}^{L}\hat{h}_l e^{-j2\pi kl/K}$
DATA DETECTION:
  $\tilde{d}_k = \mathrm{dec}(x_k/\hat{H}_k)$

4. RESULTS AND DISCUSSION

In this section, we present simulation results to demonstrate the symbol error rate (SER) improvement by using partial FFT demodulation in UWA OFDM systems. We consider an uncoded shallow water communication link operating in the 24 kHz - 36.5 kHz acoustic frequency band with a single transmit and receive element. The ratio of the relative velocity between the communicating elements v to the speed of sound $c = 1500$m/sec can reach values in excess of 10^{-3} in an underwater system and hence the received signal is normally resampled using a coarse estimate of the ratio v/c. The residual Doppler scaling either due to resampling errors or due to drifting transducers is assumed to be of the order of 10^{-4}. The OFDM system considered has $K = 1024$ subcarriers operating in a bandwidth of $B = 12.5$kHz. The subcarrier spacing is thus $\Delta f = 12.2$Hz and the symbol duration T is around 82ms. The maximum delay spread is 5 ms and the guard interval is set at $T_g = T/8$ to ensure no inter-symbol interference at the receiver. The first few data symbols are designated as pilots to train the RLS algorithm. Every fourth symbol is also designated as a pilot for the recursive estimator while the model based estimator has

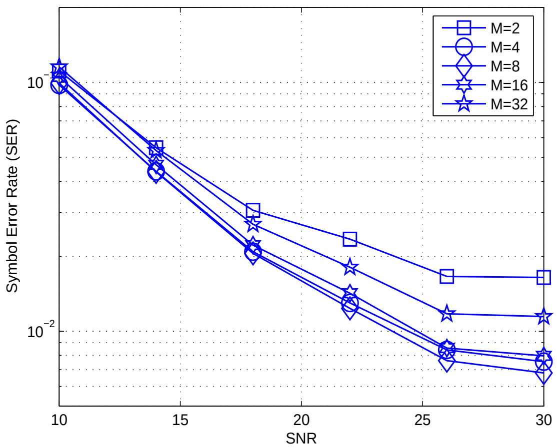

Figure 1: Performance of data detection using recursive weight estimation for $K = 1024$ subcarriers.

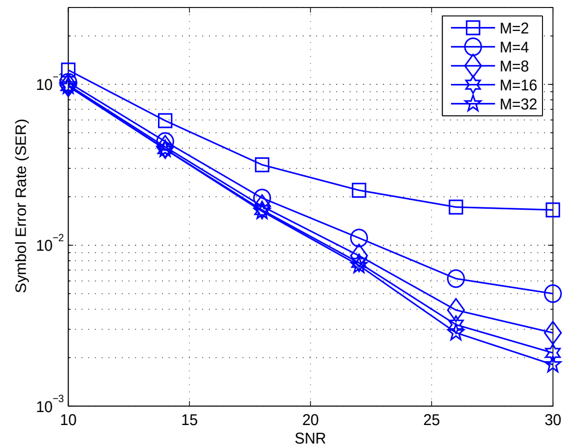

Figure 2: Performance of data detection using model based weight estimation for $K = 1024$ subcarriers.

one in eight subcarriers as pilots. The channel is a multipath Rayleigh fading channel with 6 taps and random delays and a different channel realization is used for each OFDM symbol. Though this channel model is not an accurate representation of an underwater acoustic channel, it serves to demonstrate the significant performance improvements that can be obtained by the proposed algorithms.

Fig. 1 shows the SER obtained using several values of partial FFT intervals M as a function of the Signal to Noise Ratio (SNR) when using the recursive weight estimation algorithm. We observe that at moderate SNRs the SER shows a significant improvement as M increases. This is due to the fact that the demodulator outputs are properly weighed before combining, thus compensating for the ICI and enabling improved channel estimation and data detection subsequently. As the number of partial FFT outputs M further increases, the number of combiner weights to be recursively estimated also increases and results in a performance tradeoff due to increasing estimation noise. As partial FFT demodulation does not completely eliminate the ICI, an error floor is also observed at high SNRs due to the residual ICI.

Fig. 2 shows the SER for several values of M as a function of SNR when the model based estimator is used to determine the combiner weights at the receiver. It is observed that the SER improves monotonically for increasing M and for low to moderate SNR's quickly approaches the SER for a Rayleigh fading channel with no distortion. This can be attributed to the fact that the model based estimator accurately models the distortion process. For larger values of M, the performance improvement saturates and results in a tradeoff between increasing computational cost and SER improvement. The computation of the partial FFT requires $O(MK\log K)$ computations and is the first step for both the algorithms. The complexity of the remaining computations increases linearly with K and quadratically in M. Thus, the proposed methods perform ICI at a cost which linearly increases with the number of OFDM subcarriers. Also, the recursive weight estimator does not rely on any knowledge of the distortion process and is robust, while the model based estimator relies on a fairly accurate model of the Doppler distortion and hence performs better than the recursive estimator for the scenarios considered in this paper. The considered estimators operate under different assumptions and are both relevant to UWA communications.

5. CONCLUSIONS

To improve coherent detection in UWA systems with motion-induced Doppler distortion, we proposed a method in which several FFT demodulators operate over small portions of an OFDM block, which are then combined. Two algorithms to compute the combiner weights are proposed: the first does not operate with any knowledge of the distortion process and is robust while the second assumes an accurately modeled distortion and results in lower symbol error probabilities. Numerical simulations show the significant performance improvements obtained using the proposed techniques.

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